

Business Valuation Monitor

Value creation perspectives for corporate executives and the investment community April 2009

Exploring the New World of Quantitative DLOM Analyses

Back to the future?

Old world theory offers guidance on new world valuation methodology – a fresh look at a rediscovered approach that may help quantify discounts for lack of marketability in closely held or restricted shares.

Francis Longstaff – sounds like the name of a 15th century explorer, doesn't it? Well, in a way, it is. Mr. Longstaff, in 1995, when the rest of us were still in the "Old World" of pre-IPO and restricted stock studies, issued a paperⁱ describing lack of marketability as the loss of an option to sell the stock at the time of the holder's choosing.

In today's increasingly quantitative world, business valuers are rediscovering Longstaff's model and are using it as one way to support the still largely qualitative judgment regarding the magnitude of a discount for lack of marketability (DLOM).

Longstaff's seminal article posits that option pricing theory can help explain the wide dispersion of observed DLOMs and that option pricing models can help quantify a DLOM for a specific security. We know from restricted stock studies that DLOMs vary greatly. Two examples are displayed below.

[continued>](#)

Study	Range of DLOM
Institutional Investors Study Report (1971) ⁱⁱ	-15% to 80%
Bajaj, et al. (2001) ⁱⁱⁱ	-14% to 68%

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Key variables	Effect	Study			
A. Volatility Factors					
1. Stability					
Price Volatility	Direct	MP		FMV	Bajaj
Earning Volatility	Direct	MP			
Revenue Volatility	Direct	MP			
Financial Distress	Direct				Bajaj
2. Size					
Revenue	Inverse	MP	SEC/IIS	FMV	Silber
Level of Earnings	Inverse	MP	SEC/IIS	FMV	Silber
Market Price per Share/Market Cap	Inverse	MP		FMV	
Assets	Inverse			FMV	
B. Liquidity/Holding Period					
Block Size to Shares Outstanding	Direct	MP		FMV	Silber Bajaj
Dollar – Trading Volume	Inverse	MP			

MP = Management Planning
 FMV = FMV Opinions, Inc.
 Bajaj = Mukesh Bajaj, et al.
 SEC/IIS = SEC/Institutional Investor Study
 Silber = William Silber

What factors cause this large disparity in discounts? If you look closely at the restricted stock studies, they point you to the conclusion Longstaff reached long ago – that DLOM is largely a function of an asset’s volatility and the length of time marketability is restricted. The table above groups the factors identified by the restricted stock studies into two categories: volatility and holding period. When the studies are viewed in this light, using option pricing methodologies to estimate DLOM makes perfect sense.

Recognizing that volatility and holding period are the key DLOM determinants, Longstaff derived a model that measures the loss in value resulting from the opportunity cost imposed by the period of time during which the asset is not liquid.^{iv} The model is as follows:

Longstaff’s model utilizes the following assumptions:

1. Asset liquidity is complete and continuous
2. The market is frictionless and has a constant risk-free rate
3. Investors have perfect market time and can sell the asset and can immediately invest the proceeds into a riskless asset.

Since these assumptions are not reflective of the “real world”, Longstaff’s model results in the “maximum” DLOM for a given restriction period and a given volatility. As with all of the quantitative DLOM models^v, Longstaff’s model should be used with care. All such models address relatively short-lived, publicly-traded options; all have limitations when longer restriction periods are present.

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Maximum Discount =

$$V \left(2 + \frac{\sigma^2 T}{2} \right) N \left(\frac{\sqrt{\sigma^2 T}}{2} \right) + V \sqrt{\frac{\sigma^2 T}{2\pi}} \exp \left(-\frac{\sigma^2 T}{8} \right) - V$$

V = Current value of a security that is continuously traded in a frictionless market
 σ = Standard deviation of the company’s daily stock returns (annualized)
 T = Length of time that the shares are illiquid
 N = Cumulative normal distribution function

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The table below displays the DLOMs calculated using Longstaff's model for a range of stock return volatilities and time periods.

Time of Illiquidity	Maximum Percentage Discounts for Lack of Marketability by Volatility		
	0.10	0.20	0.30
Six months	5.8%	11.8%	18.1%
One year	8.2	17.0	26.3
Two years	11.8	24.6	38.6
Five years	19.1	41.0	65.8

These outcomes provide two important conclusions:

1. For a given restriction period, the DLOM will vary greatly, depending on the stock's volatility.
2. DLOMs are not a linear function of time. The greatest risk and therefore the largest increase in the percentage discount occur early in the restriction period.

Longstaff's model results in DLOMs that are consistent with the ranges found in the restricted stock studies and consequently provide a framework for evaluating DLOMs over the displayed volatilities and holding periods.

In our practice, we use Longstaff's model to help frame DLOMs for securities with short holding periods (e.g., Rule 144 restrictions that will lapse soon after the valuation date or when performing a 409A valuation for a company that is presumed to have a successful IPO in the foreseeable future).

If you use Longstaff's model, will it lead you to the Promised Land? No, but it is a good place to start or continue your exploration into the Brave New World of quantitative DLOM analyses. •

ⁱ Longstaff, Frances A. "How Much Can Marketability Affect Security Values?" *The Journal of Finance*, Vol. 50, No. 5, December 1995.

ⁱⁱ SEC. 1971. "Discounts Involved in Purchases of Common Stock." *Institutional Investors Study Report of the Securities and Exchange Commission*. Securities and Exchange Commission, H.R. Doc. No. 64, Part 5, 92nd Congress.

ⁱⁱⁱ Bajaj, M., D.J. Denis, S.P. Ferris, and A.Sarin. 2001. "Firm Value and Marketability Discounts." *Journal of Corporation Law*, Vol. 27, No. 1.

^{iv} Dyl, Edward A. and Jiang, George J. "Valuing Illiquid Common Stock." *Financial Analysts Journal*, Volume 64, Number 4. CFA Institute. 2008.

^v Stockdale, John J. "A Test of DLOM Computational Models." *Business Valuation Review*, Volume 27, Number 3. American Society of Appraisers. Fall 2008.



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